NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES
SCHOOL OF NATURAL AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

QUALIFICATION: BACHELOR OF SCIENCE HONOURS IN APPLIED MATHEMATICS	
QUALIFICATION CODE: 08BSHM	**LEVEL: 8**
COURSE CODE: ADC801S	**COURSE NAME:** ADVANCED CALCULUS
SESSION: JULY 2023	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS: 87**

SUPPLEMENTARY/SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	**Prof A.S Eegunjobi**
MODERATOR	Prof O.D Makinde

INSTRUCTIONS
1. Answer ALL the questions. 2. Write clearly and neatly. 3. Number the answers clearly.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

1. (a) Determine the minimum distance between the origin and the hyperbola defined by $x^2 + 8xy + 7y^2 = 226$ (6)

   (b) Show that $\nabla \cdot (\nabla g^m) = m(m+1)g^{m-2}$, if $\bar{g} = xi + yj + zk$. (9)

   (c) A material body's geometric representation is a planar area R, delimited by the curves $y = x^2$ and $y = \sqrt{2 - x^2}$ within the boundaries $0 \leq x \leq 1$. The density function associated with this model is denoted as $\rho = xy$.

   i. Find the mass of the body. (4)

   ii. Find the coordinates of the center of mass. (5)

   (d) Determine the flux of $\bar{\mathbf{F}} = i - j + xyzk$ through the circular region S obtained by cutting the sphere $x^2 + y^2 + z^2 = 4$ with a plane $y = x$. (6)

   (e) Find the volume of the solid region bounded above the paraboliod $z = 1 - x^2 - y^2$ and below the plane $z = 1 - y$. (6)

2. (a) if $Q = \log(\tan x + \tan y + \tan z)$, show that

$$\frac{\sin 2x}{2}\frac{\partial u}{\partial x} + \frac{\sin 2y}{2}\frac{\partial u}{\partial y} + \frac{\sin 2z}{2}\frac{\partial u}{\partial z} = 1$$

(5)

   (b) If $x = r\cos\theta$ and $y = r\sin\theta$, find the (r, θ) equations for ϕ which obeys Laplace's equation in two-dimensional caresian co-ordinates

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

(5)

   (c) If $\mathbf{A}, \mathbf{B}$ and C are vectors, show that

$$\frac{d}{dt}\mathbf{A} \cdot \mathbf{b} \times \mathbf{C} = \frac{d\mathbf{A}}{dt} \cdot \mathbf{B} \times \mathbf{C} + \mathbf{A} \cdot \frac{d\mathbf{B}}{dt} + \mathbf{A} \cdot \mathbf{B} \times \frac{d\mathbf{C}}{dt}$$

(5)

3. (a) Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ by taking the starting from the point $\mathbf{X_1} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$ using Davidon–Fletcher–Powell (DFP) method with

$$[B_1] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \epsilon = 0.01$$

(10)

   (b) Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ by taking the starting from the point $\mathbf{X_1} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$, by using Newton's Method (10)

4. (a) Evaluate the integral

$$\int_0^{\frac{\pi}{2}} \frac{\cos^2 x dx}{(2\cos x + \sin x)^2} \quad \text{given} \quad \int_0^{\frac{\pi}{2}} \frac{\cos x dx}{\alpha \cos x + \sin x} = \frac{\alpha\pi}{2(\alpha^2 + 1)} - \frac{\ln \alpha}{\alpha^2 + 1}$$

(8)

(b) Find the maximum possible volume of a rectangular box that is completely enclosed by the surface of the ellipsoid defined by the equation $2x^2 + 3y^2 + z^2 = 18$, where each of its edges is parallel to one of the coordinate axes. (8)

End of Exam!